

Reliability-Based Interaction Curves For reinforced Concrete Design of Short Columns to Eurocode 2

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Abstract

The work presented in this paper examined the criteria of Eurocode 2 (EC 2) (2004) for the design of reinforced concrete short columns subjected to axial loads and bending moments using First Order Reliability method (FORM). Individual design variables of the columns were considered random with known probability distributions. Computations of design safety indices were carried as described by Low and Tang (2007). Interaction curves were plotted considering varying safety indices of the columns. The choice of a target reliability index was made to correspond with values recommended by Joint Committee on Structural Safety (JCSS, 2001). A design example was included to demonstrate the applicability of the developed reliability-based interaction as against the current EC 2 design charts. It was shown that considering the same loading and geometrical conditions of the column, the reliability-based procedure gave higher steel reinforcements at target safety indices of 2.5, 3.0, 4.0 and 5.0; and are safer than the deterministic design.

Keywords

Eurocode 2; Reinforced Concrete Short Columns; Interaction Curves; Target Safety Index; Reliability-Based Design

Introduction

Columns are compression members which transmit compressive forces from one part of a structure to another. The most well-known form of a column is a straight strut with axial compressive forces applied on the member ends. Columns are vertical members with large length-to-depth (L/D) ratios subjected to predominantly compressive loads, and in some cases, columns may be subjected to significant bending. The strength of a column cross-section can be determined from geometry of the cross-section, the constitutive relationships of the concrete and steel, as well as consideration of equilibrium and strain compatibility. The strength is usually expressed in the form of a load-moment strength interaction diagram which plots the locus of ΦM_u versus ΦN_u values, where M_u is the ultimate strength in bending at a cross-section of an eccentrically loaded compression member, N_u is the corresponding ultimate strength in compression at the same cross-section of the eccentrically loaded compression member, and Φ is the strength reduction factor to account for variability in geometry and material properties (Russell and Andrew, 2000; Mustapha, 2014).

In practice, the longitudinal steel in a reinforced concrete column is chosen among other methods, with the aid of an interaction diagram. The interaction diagram is a graphical summary of the ultimate bending capacity of a range of reinforced concrete columns with different dimensions and areas of longitudinal reinforcement.

The development of design charts for column sections therefore provides structural designers with an alternative way to design such column sections more easily, accurately, and in turn, provide greater safety to the structure being designed (Bernardo, 2007; Bouchaboub and Samai, 2013).

In probabilistic assessment, any uncertainty about a design variable (expressed in terms of its probability density function) is explicitly taken into account in the evaluation of effects of uncertainties associated with the variable. The study of structural reliability is therefore concerned with the calculation and prediction of the probability of limit state violation for engineering structures at any stage during their live times (Melchers, 1999).

Hence reliability-based design interaction curves were developed for symmetrically reinforced concrete short columns in accordance with requirements of EC 2, using FORM, which allows a systematic consideration of all the uncertainties involved in the design process (Gollwitzer et al., 1988). The interaction curves were plotted for the column sections at pre-defined safety levels. The reliability-based design interaction curves would guide designers on the knowledge of the expected levels of safety of the sections being designed.

Methodology

Capacity of Short Columns

Short columns usually fail by crushing. The Euler's critical load for a pin-ended strut is given as (Mosley, et al., 2007; Reynolds, et al., 2008):

$$N_{crit} = \frac{\pi^2 EI}{l^2} \quad (1)$$

The crushing load, N_{ud} of a truly axially loaded column may be taken as (EC 2):

$$N_{ud} = 0.567f_{ck}A_c + 0.87A_s f_{yk} \quad (2)$$

In equations (1) and (2), π is a constant (equals to 22/7), E is the modulus of elasticity in N/mm², I is the second moment of area of the section in mm⁴, L is the effective column height in mm, f_{ck} and f_{yk} are the characteristic compressive cylinder strength of concrete at 28 days and characteristic yield strength of steel respectively (both in N/mm²), and A_c and A_s are the cross-sectional areas of the column section and longitudinal steel respectively (both in mm²).

First Order Reliability Method

Probabilistic design is concerned with the probability that a structure will realize the functions assigned to it. If R is the strength capacity and S the loading effect(s) of a structural system which are functions of random variables. The main objective of reliability analysis of any system or component is to ensure that R is never exceeded by S . In order to investigate the effect of the variables on the performance of a structural system, a limit state equation in terms of the basic design variable is required (Melchers, 1999; Abubakar, et al., 2014). The limit state equation is referred to as the performance or state function and is expressed as:

$$g(\mathbf{X}) = g(X_1, X_2, \dots, X_n) = R - S \quad (3)$$

Where X_i ; $i = 1, 2, \dots, n$, represent the basic design variables.

The limit state of the system can then be expressed as:

$$g(\mathbf{X}) = 0 \quad (4)$$

Graphically as shown in Figure 1, the line $g(\mathbf{X}) = 0$ represents the failure surface, while $g(\mathbf{X}) > 0$ represents the safe region, and $g(\mathbf{X}) < 0$ corresponds to the failure region.

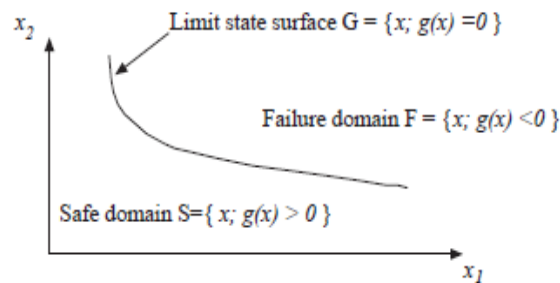


FIGURE 1: FAILURE DOMAIN, LIMIT STATE SURFACE AND SAFE DOMAIN (THOFT-CHRISTENSEN AND BAKER 1982)

Limit State Function

The calculation of the limit state function is performed for discrete combination of basic variables of the reinforced concrete short columns in accordance with EC 2 and is expressed in terms of the variables as:

$$g(X) = (A + B + C) - D \quad (5)$$

In which;

$$A = \left(\frac{N}{bh f_{ck}} \right) \left\{ 0.5 - 0.4 \left(\frac{x}{d} \right) \left(\frac{d}{h} \right) \right\}, B = (0.5 P \frac{f_{sc}}{f_{ck}} \{ 0.4 \left(\frac{x}{d} \right) \left(\frac{d}{h} \right) - 1 + \left(\frac{d}{h} \right) \}), C = (0.5 P \frac{f_{st}}{f_{ck}} \{ \frac{d}{h} - 0.4 \left(\frac{x}{d} \right) \left(\frac{d}{h} \right) \}), \text{ and } D = \left(\frac{M}{bh^2 f_{ck}} \right)$$

It is to be noted from Equation (5) that the terms (A+B+C) and D correspond to the resistance of the member, R, and its loading effect, S, respectively as given in Equation (3). Therefore, the limit state function is given by

$$g(X) = \left(\frac{N}{bh f_{ck}} \right) \left\{ 0.5 - 0.4 \left(\frac{x}{d} \right) \left(\frac{d}{h} \right) \right\} + (0.5 \rho \frac{f_{sc}}{f_{ck}} \{ 0.4 \left(\frac{x}{d} \right) \left(\frac{d}{h} \right) - 1 + \left(\frac{d}{h} \right) \}) + (0.5 P \frac{f_{st}}{f_{ck}} \{ \frac{d}{h} - 0.4 \left(\frac{x}{d} \right) \left(\frac{d}{h} \right) \}) - \left(\frac{M}{bh^2 f_{ck}} \right) \quad (6)$$

Where in equation (6), N and M are the applied axial loading and bending moment respectively, f_{sc} and f_{st} are the compressive and tensile stresses in the steel reinforcement respectively, b is the breadth of the section, h is the overall depth of the section, ρ is the steel reinforcement ratio, $\frac{d}{h}$ is the reinforcement position and $\frac{x}{d}$ is the position of neutral axis. The coefficients of variations of the design variables were obtained from literature (Mirza, et al., 1979; Arafah, 1997; Phoon, 2005; Mirza, 2011).

Results and Discussions

Reliability-Based Interaction Curves

The reliability-based interaction curves were developed considering varying values of target safety indices, as well the design parameters of the short columns given in equation (6). The FORM procedure adopted in the study was given by Low and Tang (2007). The developed reliability-based charts are as shown in Figures (2) to (11).

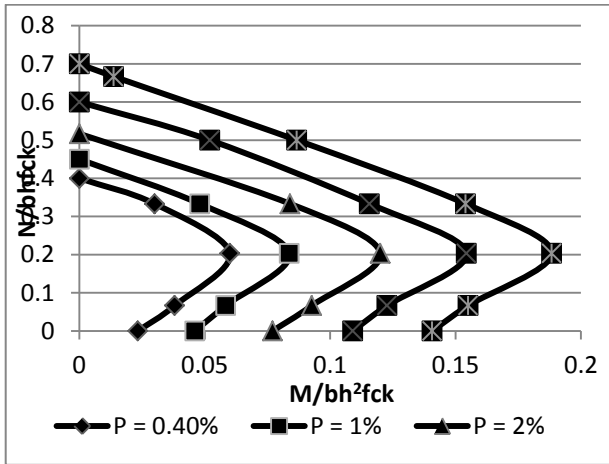


FIGURE 2: RELIABILITY BASED COLUMN DESIGN CHART FOR $B_r = 2.5$, $D/H = 0.8$

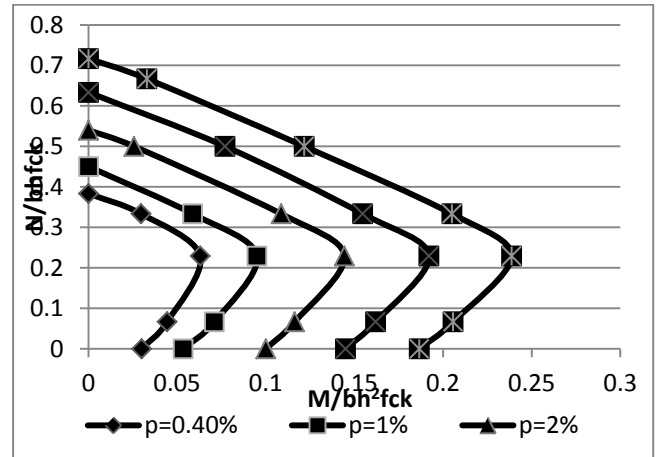


FIGURE 3: RELIABILITY BASED COLUMN DESIGN CHART FOR $B_r = 2.5$, $D/H = 0.9$

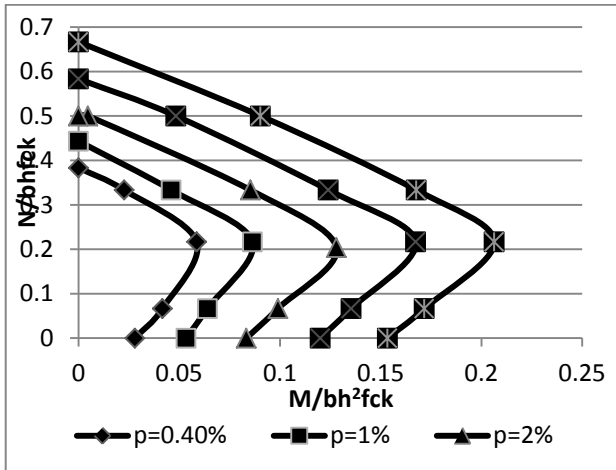


FIGURE 4: RELIABILITY BASED COLUMN DESIGN CHART FOR $B_r = 3$, $D/H = 0.85$

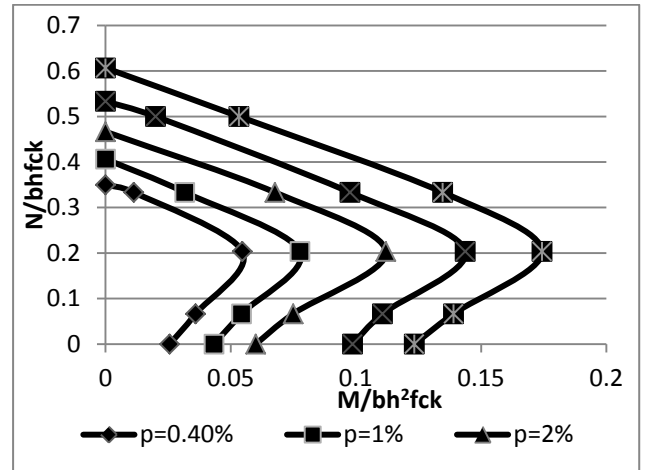


FIGURE 5: RELIABILITY BASED COLUMN DESIGN CHART FOR $B_r = 3.5$, $D/H = 0.8$

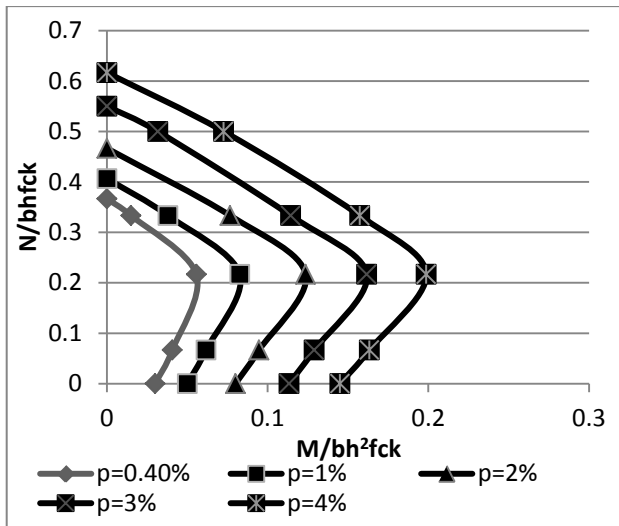


FIGURE 6: RELIABILITY BASED COLUMN DESIGN CHART
FOR $B_r = 3.5$, $D/H = 0.85$

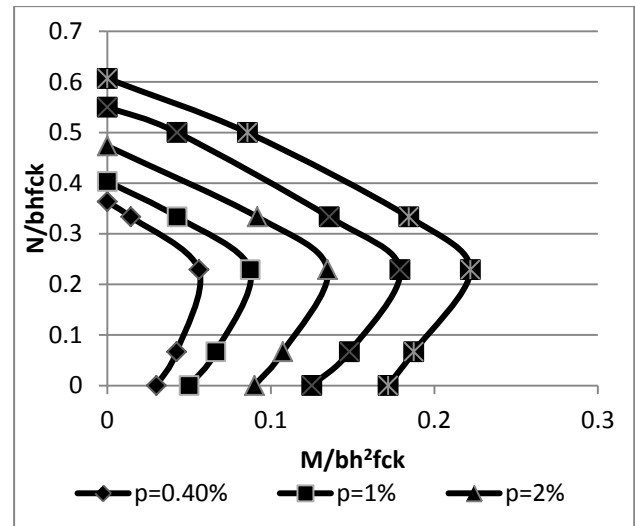


FIGURE 7: RELIABILITY BASED COLUMN DESIGN CHART
FOR $B_r = 3.5$, $D/H = 0.9$

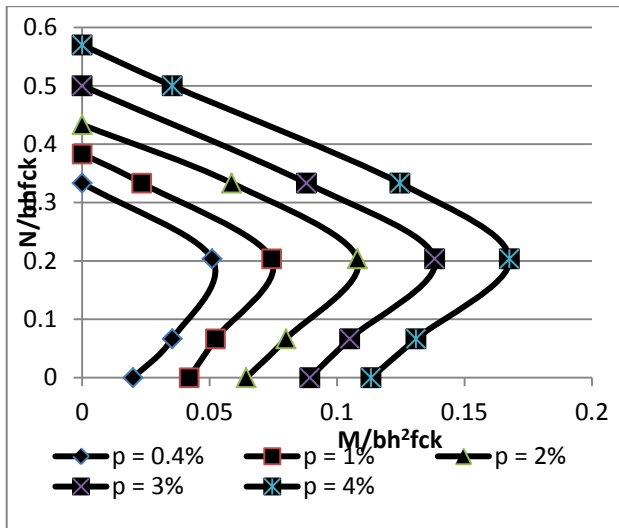


FIGURE 8: RELIABILITY BASED COLUMN DESIGN CHART
FOR $B_r = 4.0$, $D/H = 0.8$

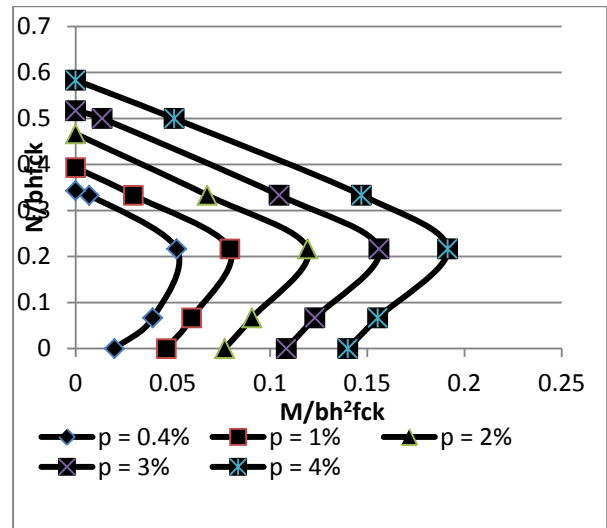


FIGURE 9: RELIABILITY BASED COLUMN DESIGN CHART
FOR $B_r = 4.0$, $D/H = 0.85$

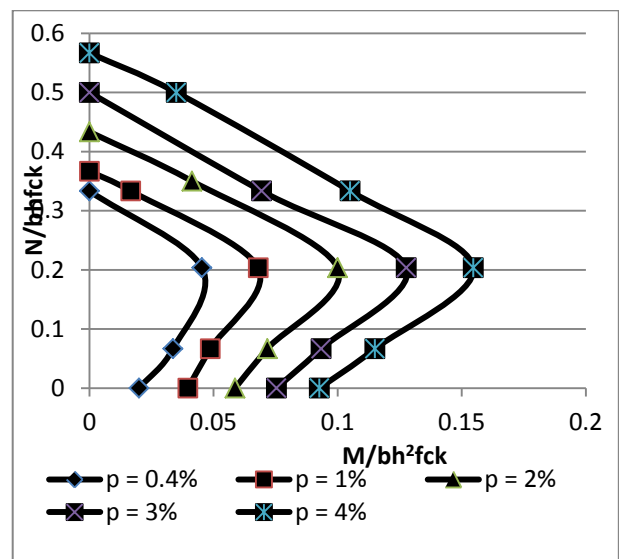


FIGURE 10: RELIABILITY BASED COLUMN DESIGN CHART
FOR $B_r = 5.0$, $D/H = 0.8$

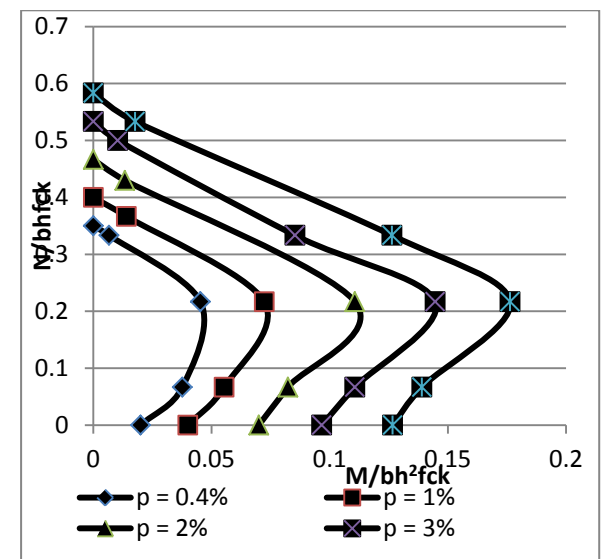


FIGURE 11: RELIABILITY BASED COLUMN DESIGN CHART
FOR $B_r = 5.0$, $D/H = 0.85$

Design Example

An internal column of a multi-storey building with dimensions 275 x 300mm and 3500 mm in height, was subjected to an ultimate axial load, N of 1600 kN and a bending moment, M of 60 kNm including effect of imperfections. The column is to be designed assuming that the characteristic strengths of concrete (f_{ck}) and steel (f_{yk}) are 25N/mm² and 500 N/mm² and deviation allowance, ΔC_{dev} of 5 mm. Assuming also that the diameter of longitudinal bars (Φ) = 25 mm, minimum cover to main steel for bond, $c_{min,b}$ = 25 mm and diameter of links, (Φ') = 8 mm.

It is required to determine the area of steel required using:

- EC 2 (2004) design procedure (charts);
- reliability-based interaction chart at a target safety index of 2.5;
- reliability-based interaction chart at a target safety index of 3.0;
- reliability-based interaction chart at a target safety index of 4.0; and
- reliability-based interaction chart at a target safety index of 5.0

Eurocode 2 (2004) Design

Minimum eccentricity, $e_0 = h/30 = 300/30 = 10\text{mm} \geq 20\text{mm}$ (EC 2)

Minimum design moment = $e_0 N = 20 \times 10^{-3} \times 1.6 \times 10^3 = 32 \text{ kNm} < M$

Since $M = 60 \text{ kNm}$ and assuming $\lambda < \lambda_{lim}$.

Minimum cover to links for exposure class XC1, $c_{min,dur} = 15 \text{ mm}$ (EC 2).

And the nominal cover;

$c_{nom} = c_{min,b} + \Delta C_{dev} = 25 + 5 = 30 \text{ mm} \Rightarrow$ Minimum cover to links = $c_{nom} - \Phi' - \Delta C_{dev} = 30 - 8 - 5 = 17 \text{ mm} > c_{min,dur} = 15 \text{ mm}$ OK.

Therefore, $d_2 = 30 + 25/2 = 42.5 \text{ mm}$, $d_2/h = 42.5/300 = 0.141$; $d/h = 257.5/300 = 0.85$.

Round up to 0.15 and use EC 2 (2004) column chart with $d_2/h = 0.15$.

It follows also that; $\frac{N_{Ed}}{bh f_{ck}} = 1.6 \times 10^6 / 275 \times 300 \times 25 = 0.78$, $\frac{M_{Ed}}{bh^2 f_{ck}} = 60 \times 10^6 / 275 \times 300^2 \times 25 = 0.097$

The minimum area of longitudinal steel (EC 2) is $A_{smin} = 0.10N / 0.87f_{yk} \geq 0.002A_c$; 367.8mm^2

From EC 2 (2004) design chart, $\frac{A_s F_{yk}}{bh f_{ck}} = 0.25$

Area of steel required is $A_s = 0.25 \times bh f_{ck} / f_{yk} = 0.25 \times 275 \times 300 \times 25 / 500 = 1031.25\text{mm}^2$.

Therefore design is therefore satisfactory.

Reliability-Based Design at a Target Safety Index of 2.5

From Figure 2, it follows that; $\frac{A_s F_{yk}}{bh f_{ck}} = 0.4$

Area of steel required is $A_s = 0.4 \times bh f_{ck} / f_{yk} = 0.4 \times 275 \times 300 \times 25 / 500 = 1650\text{mm}^2$.

Reliability-Based Design at a Target Safety Index of 3.0

From Figure 4, it follows that; $\frac{A_s F_{yk}}{bh f_{ck}} = 0.4$

Area of steel $A_s = 0.4 \times bh f_{ck} / f_{yk} = 0.4 \times 275 \times 300 \times 25 / 500 = 1650\text{mm}^2$.

Reliability-Based Design at a Target Safety Index of 4.0

From Figure 9, it follows that; $\frac{A_s F_{yk}}{bh f_{ck}} = 0.6$

Area of steel $A_s = 0.6 \times bhf_{ck}/f_{yk} = 0.6 \times 275 \times 300 \times 25/500 = 2475\text{mm}^2$

Reliability-Based Design at a Target Safety Index of 5.0

From Figure 11, it follows that; $\frac{A_s F_{yk}}{bh f_{ck}} = 0.7$

Area of steel $A_s = 0.7 \times bhf_{ck}/f_{yk} = 0.7 \times 275 \times 300 \times 25/500 = 2887\text{mm}^2$

TABLE 1 DESIGN RESULTS FOR SHORT COLUMN

S/No.	Chart type	$\frac{A_s F_{yk}}{bh f_{ck}}$	Area of Steel Required A_s (mm ²)
1	EC 2 (2004) design charts	0.25	1031.25mm ² .
2	Reliability-based design (target safety index = 2.5)	0.4	1650mm ²
3	Reliability-based design (target safety index = 3.0)	0.4	1650mm ² .
4	Reliability-based design (target safety index = 4.0)	0.6	2475mm ²
5	Reliability-based design (target safety index = 5.0)	0.7	2887mm ²

From the results presented in Table 1, it can be observed that for the same loadings and geometrical conditions of the column, the design results for target safety indices of 2.5 and 3.0 are the same. However, at target safety indices of 4.0 and 5.0, there was an increment in the area of steel required by of 50% and 43% respectively, when compared to target safety index of 3.0.

Also, the design in accordance with EC 2 gave lower area of reinforcement than the reliability-based design. The implied safety index of the EC 2 design (which is equal to 1.57) shows that the designed section will fail under ultimate limit state conditions.

Conclusion

Reliability-based design concepts considering Eurocode 2 design requirements and target safety index requirements of JCSS (2001) were used to develop interaction charts for reinforced concrete short columns.

The developed interaction charts have a major advantage over deterministic charts by systematic adjustment of design variables to reflect the consequences of failure because of the explicit probabilistic treatment given to uncertain variables in the design equations. It was shown that considering the same loading and geometrical conditions of the column, the design results for target safety indices of 2.5 and 3.0 were found to be the same. However, at target safety indices of 4.0 and 5.0, there was an increment in the area of steel required by of 50% and 43% respectively, when compared to target safety index of 3.0.

Also, the design in accordance with EC 2 gave lower area of reinforcement when compared with the reliability - based design at target safety indices of 2.5, 3.0, 4.0 and 5.0; but there is however compromise to safety of the section.

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